Wind Vector Reconstruction Method for Molas B300

1 Introduction

Molas B300 is a ground based wind doppler lidar. The lidar uses the velocity-azimuth display (VAD) scan pattern to measure vector winds. As is shown in Figure 1, 4 independent LOS are achieved by scanning the beam in azimuth at a fixed elevation angle (28°).

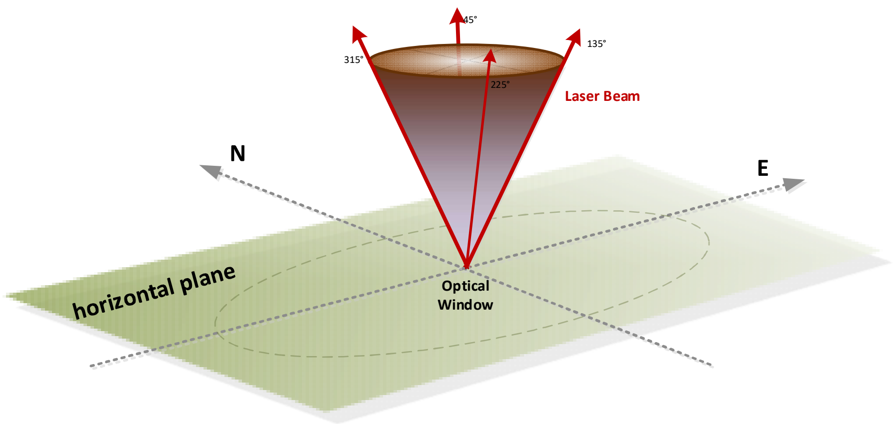

Figure 1 Scanning Pattern of Lidar

2 Reconstruction Method

For beam i, the radial wind speed can be calculated from the vector wind $\vec{V} = [u, v, w]^T$, that is

$$\hat{v}_{los,i} = \vec{n}_i \cdot \vec{V}$$

$\vec{n}_i$ is the unit vector of LOS i in inertial coordinate system, $i = 0,1,2,3$, and its equation is

$$\vec{n}_i = [sin\gamma cos\theta_i, sin\gamma sin\theta_i, cos\gamma]$$

with the elevation angle $\gamma = 28^{\circ}$, the azimuth $\theta_i = 45^{\circ} + i \cdot 90^{\circ}$.

Combine the equation of 4 beams, we can get

$$\begin{bmatrix} \hat{v}_{los,0} \\ \hat{v}_{los,1} \\ \hat{v}_{los,2} \\ \hat{v}_{los,3} \end{bmatrix} = \begin{bmatrix} sin\gamma cos\theta_0 & sin\gamma sin\theta_0 & cos\gamma \\ sin\gamma cos\theta_1 & sin\gamma sin\theta_1 & cos\gamma \\ sin\gamma cos\theta_2 & sin\gamma sin\theta_2 & cos\gamma \\ sin\gamma cos\theta_3 & sin\gamma sin\theta_3 & cos\gamma \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

The lidar measures the radial wind speed $v_{los,i}$. For a uniform wind field, the measured radial wind speed $v_{los,i}$ must be close to the calculated ones $\hat{v}_{los,i}$. So, their square error must be minimized, that is

$$L_s = \frac{1}{2}\sum_{i=0}^{3}\left(\hat{v}_{los,i} - v_{los,i}\right)^2$$

The minima of the above equation can be solved with LSE or LM method. Then the vector wind $\vec{V} = [u, v, w]^T$ can be obtained.

Let $\mathbf{x} = \begin{bmatrix} v_{los,0} \\ v_{los,1} \\ v_{los,2} \\ v_{los,3} \end{bmatrix}$, $\mathbf{H} = \begin{bmatrix} sin\gamma cos\theta_0 & sin\gamma sin\theta_0 & cos\gamma \\ sin\gamma cos\theta_1 & sin\gamma sin\theta_1 & cos\gamma \\ sin\gamma cos\theta_2 & sin\gamma sin\theta_2 & cos\gamma \\ sin\gamma cos\theta_3 & sin\gamma sin\theta_3 & cos\gamma \end{bmatrix}$, $\boldsymbol{\theta} = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$, then

$$L_s(\boldsymbol{\theta}) = (\mathbf{x} - \mathbf{H}\boldsymbol{\theta})^T(\mathbf{x} - \mathbf{H}\boldsymbol{\theta})$$

Let the gradient of the equation be 0, we can get the LSE of the estimation,

$$\frac{\partial L_s(\boldsymbol{\theta})}{\partial \boldsymbol{\theta}} = -2\mathbf{H}^T\mathbf{x} + 2\mathbf{H}^T\mathbf{H}\boldsymbol{\theta} = 0$$

$$\boldsymbol{\theta} = \begin{bmatrix} u \\ v \\ w \end{bmatrix} = (\mathbf{H}^T\mathbf{H})^{-1}\mathbf{H}^T\mathbf{x}$$

The wind speed and wind direction can be calculated with the following equation:

$$\begin{cases} ws = \sqrt{u^2 + v^2} \\ wd = arctan2(v, u) \end{cases}$$